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AUTHOR De Corte, Erik; Verschaffel, Lieven
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ABSTRACT

Recent research on solving addition and subtraction word problems has resulted in the construction of theoretical models of children's problem-solving processes. Some of these models have been translated into computer programs. Characteristics and predictions of the theoretical analysis developed by Riley, Greeno, and Heller (1983) are discussed in this paper, and tested in a study with 30 first graders. They were administered a series of Piagetian, memory, and counting tasks, plus eight addition and subtraction word problems, in interviews conducted with individuals three times during the 1981-82 school year. Six student protocols are included to illustrate the findings. A number of results confirm several predictions and implications of the computer mode. However, some important findings are not in agreement with these models. These specific findings, concerning answer patterns, appropriate problem representations, correct solution strategies, and the nature of errors, are discussed. (MNS)

AN EMPIRICAL VALIDATION OF COMPUTER MODELS OF CHILDREN'S WORD PROBLEM SOLVING

Erik DE CORTE and Lieven VERSCHAFFEL (1)

Center for Instructional Psychology

University of Leuven, Belgium

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Corresponding address : Erik De Corte, Center for Instructional Psychology, Department of Education, University of Leuven, Vesaliusstraat 2, B-3000 Leuven, Belgium.

(1) Lieven Verschaffel is Senior Research Assistant of the National Fund for Scientific Research, Belgium.

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Abstract

Recent research on problem solving of addition and subtraction word problems has resulted in the construction of theoretical models of children's problem-solving processes. Some of these models have been translated in computer programs. In the present paper characteristics and predictions of the theoretical analysis developed by Riley, Greeno & Heller (1983) are discussed and tested using empirical data collected in a recent longitudinal study with first graders. It will be shown that although a number of results confirm several predictions and implications of the computer models, there are also important findings that are not in agreement with these models.

1. Introduction

Over the past few years a considerable number of studies has been undertaken that try to assess and to get a better understanding of young children's problem solving with respect to simple addition and subtraction word problems. This research has resulted in the construction of theoretical models of children's problem-solving processes. Some of these models have been translated in computer programs (Briars & Larkin, 1982; Riley, Greeno & Heller, 1983). These computer models simulate the knowledge and actions that are required to understand and solve addition and subtraction word problems. On the basis of such models predictions can be made concerning the level of difficulty of different types of word problems, the strategies applied to solve them, and the errors that one can expect as a consequence of specific shortcomings in children's knowledge and actions. One could wonder why some researchers translate their view of young children's word problem solving in a computer program. Briars & Larkin (1982) justify the use of computer simulation as follows :

> "The rules comprising an information-processing model are commonly written in computer language to form a computer program, and that is the case with our model. The reason is that human beings are complex, and even rudimentary models of their behavior must reflect this complexity. Even a simple model has many rules that interact in complex ways, making tracing the model's performance by hand difficult if not impossible. The computer is a tool for accurately implementing such models to make predictions.
>
> None of the preceding discussion means that human beings "are just like big computers", or that simple information-processing models capture all the richness of human intelligence. But computers do provide the best modelling tool we know for capturing with useful precision the complex interaction of laws we believe must be involved in human performance".

Using qualitative empirical data collected by self-reporting techniques such as thinking aloud and retrospection, and by systematic behavior observation, the adequacy and the validity of the computer models can be tested. In the present paper this will be done with respect to the theoretical analysis with computer programs developed by Riley et al. (1983). First, we will give an overview of this theory. Then some predictions that derive from the theory will be confronted with the findings of a recent empirical study on the development of thirty first graders problem-solving skills with respect to a series of simple addition and subtraction word problems (Verschaffel 1984).

2. Riley, Greeno & Heller's theoretical analysis of word problem solving

In the seventies Greeno, in collaboration with Riley and Heller, started a series of investigations aimed at a better understanding of the problem-solving processes of simple addition and subtraction word problems (Greeno, 1980; Heller & Greeno, 1978; Riley & Greeno, 1978). These researchers argued that previous models of problem solving, such as the one proposed by Bobrow (1968), do not give an appropriate description of the solution processes of competent problem solvers. Bobrow's model is a syntactic one, in which the main process is direct translation of the verbal text into mathematical equations with minimal use of the meaningful context of the problem. In the model developed by Greeno and associates semantic processing is considered to be the main component in skilled problem solving, i.e. the problem solver "constructs a semantic network representing the information in the problem" (Greeno, 1980, p. 9).

Starting from this assumption Heller & Greeno (1978) have analyzed a large number of simple addition and subtraction word problems in textbooks and achievement tests. This led them to the identification of three different schemata underlying those problems. These schemata which specify alternative structures of quantitative information, are : change, combine, and compare. Consequently three types of problems can be distinguished.

The change schema consists of a start set, a transfer set, and a result set; it is used for understanding situations in which some event changes the value of a quantity; for example : "Joe has 3 marbles; Tom gives him 5 more marbles; how many marbles does Joe have now ?" The combine schema is composed of two subsets and one superset; it is used to represent static situations where there are two amounts that are considered either separately or in combination, as in the following case : "Joe has 3 marbles; Tom has 5 marbles; how many marbles do they have altogether?" The compare schema consists of a compared set, a referent set, and a difference set; it is used for understanding problems that involve two amounts that are compared and the difference between them, such as this problem : "Joe has 3 marbles; Tom has 5 more marbles than Joe; how many marbles does Tom have?" (Greeno, 1980, 1982).

Within each of the three problem types Heller & Greeno (1978) made further distinctions depending on the identity of the unknown quantity; the change and the compare problems were also subdivided depending on the direction of the event or the relation, such as an increase or decrease in a change problem. As a result they obtained fourteen different types of addition and subtraction problems of which Table 1 gives an overview.

Insert Table 1 about here

In a more recent contribution Riley, Greeno & Heller (1983) have analyzed the development of children's problem-solving skills on such word problems. They give an explicit description of the knowledge components underlying the solution of such problems, and also of their development in young children.

In their theoretical analyses Riley et al. (1983) distinguish three kinds of knowledge during problem solving:

> "(a) problem schemata for understanding the various semantic relations discussed earlier; (b) action schemata for representing the model's knowledge about actions involved in problem solutions; and (c) strategic knowledge for planning solutions to problems" (p. 165).

The problem schemata represent the competent problem solver's knowledge concerning the basic semantic relations underlying simple addition and subtraction problems (change, combine, and compare). This means that Riley and associates consider the three types of schemata described above, not only to be abstract structures based on a linguistic analysis of story problems; they also hypothesize that there are psychological correlates to those structures in the cognitive system of the competent problem solver and that they can be conceived as general cognitive schemata containing the subject's knowledge about increasing, decreasing, combining, and comparing groups of objects. Understanding a word problem is, then, considered to be the activation of one of these three schemata, which, in turn guides the construction of the semantic representation of the story.

Action schemata refer to the knowledge about actions such as counting, adding and subtracting, that can be applied to find the unknown quantity in a given problem. According to Riley et al. the action schemata are organized into different levels of complexity. More complex schemata are composites of more simple ones, and are comparable to the solution strategies for simple word problems described by Carpenter & Moser (1982; see also De Corte & Verschaffel, 1984). For example, to solve the problem "Joe has 8 marbles; Tom has 5 marbles; how many marbles does Joe have more than Tom?", a possible complex action schema could be composed of the following simple schemata: (1) make a set of objects corresponding to the first number in the problem, namely eight; (2) make a set corresponding to the second number in the story, namely five; (3) match, i.e. separate the largest set in two parts, one of which equals the smallest set; (4) count the remaining objects in the other part of the largest set.

By adding the third kind of knowledge, namely strategic knowledge for

planning solutions Riley et al. (1983) underline the goal-oriented and planned character of competent problem-solving activity. According to the authors

> "planning involves working out a solution from the top down, that is, choosing a general approach (e.g. match) to a problem, then deciding about actions that are somewhat more specific, and only then working out the details". (p. 170)

Riley et al. (1983) also propose a theoretical analysis of the <u>development</u> of children's problem-solving skills in which the major factor is assumed to be the acquisition of an improved ability to understand problem information. For the three semantic categories included in their study (change, combine and compare), the authors identified different levels of skill, each associated with a distinctive pattern of correct responses and errors on the various problem types within the three categories. They developed models that simulate these different levels of performance, some of which are implemented in a computer program. The main differences between these models relate to the way in which the problem information is represented and the way in which quantitative information is manipulated. Models with more detailed representational schemata and more sophisticated action schemata represent the more advanced levels of problem-solving skill, and, therefore, they can solve more problem types of a certain category. Riley et al. based their analysis largely on a rational task analysis, and also on empirical data from a developmental study with twenty children from kindergarten and twenty from the first, the second and the third grade. Those children were given individually word problems of the fourteen types in Table 1. Each time the child was asked to solve the problem using blocks.

Riley et al. (1983) give the most detailed description of their computer models for the change problems. Therefore we focus our presentation of their view on this type of problems for which they distinguish three models representing three levels of skill. With each level a different pattern of correct and wrong answers on the six types of change problems is associated, as represented in Table 2. For example, at the lowest level (level 1) three kinds of change problems are solved correctly: change 1, 2 and 4; at this level the change 3 and 5 problems are answered erroneously with the number 8 and 5 respectively, while no answer is given on the change 6 problem.

Insert Table 2 about here

Model 1 simulates the lowest level of skill and has available only the most simple action schemata (e.g. making a set containing a given number of

objects, counting the number of objects in a set), and also a simple schema for representing quantitative information as shown in Figure 1.

Insert Figur' 1 about here

When concrete objects (e.g. blocks) are available model 1'. knowledge is sufficient to solve correctly the change problems 1, 2, and 4. These three problem types share two characteristics: the action necessary to solve each problem can be found on the basis of local problem features, and the solution set is available in the external problem representation at the time the question is asked (Riley et al., 1983, p. 176). For example, model 1 solves the first problem in Table 1 ("Joe had 3 marbles; then Tom gave him 5 more marbles; how many marbles does Joe have now?") by making first a set of three blocks, then adding five more blocks, and finally counting all the blocks.

Due to its defective knowledge model 1 makes predictable errors on the more difficult change problems. Let us illustrate this first for change problem 3: "Joe had 3 marbles; then Tom gave him some marbles; now Joe has 8 marbles; how many marbles did Tom give him?" In response to the first sentence model 1 makes a set of three blocks, and using the schema shown in Figure 1 it represents this set as a quantity whose identity is Joe and amount is three. Because the second sentence does not mention a specified amount, model 1 does nothing and it also does not change its problem representation. In reaction to the third sentence model 1 creates the goal to make a set of eight blocks: it counts the three blocks that were put out before and then adds blocks until there are eight blocks. Model 1's representation at that time is given in Figure 2.

Insert Figure 2 about here

When finally the question is asked ("How many marbles did Tom give him?") model 1 counts all the blocks and gives eight as its answer. Because the start set and the change set are not distinguished in its final problem representation, model 1 interpretes the question as a request to determine the total number of marbles (Riley et al., 1983, p. 176-177).

Model 1 does neither succeed to solve the change problems 5 and 6. On problem 6 ("Joe had some marbles; then he gave 5 marbles to Tom; now Joe has 3 marbles; how many marbles did Joe have in the beginning?") the model does not give an answer at all. Because no quantity is specified, model 1 does not react to the first sentence. In response to the second sentence

the goal is created to remove five blocks; however, because there is no set available from which five blocks can be removed, model 1 stops the solution process.

Compared with model 1, model 2 has available more action schemata (for example, the counting-on schema), and also a schema to represent internally the quantities and their relations in a change problem. Using this schema model 2 can construct and retain a mental representation in which a certain position and role is attributed to the information elements in the problem situation (see Figure 3). Consequently, model 2 constructs a different problem representation than model 1.

Insert Figure 3 about here

On the basis of this analytic problem representation model 2 can identify and count the response set when asked the question "How many marbles did Tom give him?", albeit that in the external problem representation the start set and the change set are not distinguished.

Model 3 like model 2, has the change schema to represent problem situations. However, in contrast to model 2, this model can use this schema in a top-down way to construct a representation of the problem as a whole before solving it. Moreover, model 3 masters also the combine schema for representing static part-whole relations, and it is capable of transforming the original problem representation in terms of the change schema to this combine schema. Mastery of this additional knowledge components allows model 3 to solve certain change problems (e.g. change 1 and 3) more quickly and effectively, but also to solve the more difficult change problems 5 and 6. For, the theoretical analysis by Riley et al. (1983) assumes that the appropriate quantitative actions to solve change 5 and 6 problems can only be found, when the problem solver succeeds to transform the original representation as a change problem in terms of the combine schema. Figure 4 shows how model 3 re-represents the change 5 problem from Table 2 before selecting a quantitative action.

Insert Figure 4 about here

Riley et al. (1983) mention some data that support their theoretical analysis. Besides data concerning the difficulty level of the different problem types, they also report more direct evidence obtained in a study mentioned before with children from kindergarten till the third grade. The response patterns of most of these children matched one of the three

response patterns representing the three levels of skill. For example, only 9 % of the kindergarten children and 5 % of the first graders were not in accordance with one of the three models.

In the next section several hypotheses that derive from Riley et al.'s (1983) theoretical analysis of the development of problem-solving skills on simple addition and subtraction word problems will be confronted with the results of one of our investigations. This section is organized around four topics: the answer patterns of the individual pupils on change problems (1); the problem representations of the children who answer the problems correctly (2) and their solution strategies (3); the errors committed on those word problems (4).

3. Design and results of the study

3.1. Design and method

The data on children's representation and solution processes of addition and subtraction word problems were collected during the school year 1981-1982. Thirty first graders were interviewed individually three times during that school year: at the very beginning in September, in January and at the end of June. Together with a series of Piagetian tasks, memory tasks and counting tasks, they were administered each time eight elementary arithmetic word problems: four change problems (a change 1, a change 2, a change 5 and a change 6 problem), two combine problems (a combine 1 and a combine 2 problem) and two compare problems (a compare 1 and a compare 3 problem). The word problems were read aloud by the interviewer, and the children were asked to perform the following tasks with respect to each problem: (1) to retell the problem, (b) to solve it, (c) to explain and justify their solution strategy, (d) to build a material representation of the story with puppets and blocks, and (e) to write a matching number sentence. When the child was unable to solve a problem independently, the interviewer gave some assistance consisting of one or a combination of the following interventions: (1) re-reading the problem; (2) suggesting the use the puppets and the blocks; (3) pointing out a counting error or an error in carrying out an arithmetic operation. If the child still did not find the answer, the interviewer switched over to the so-called systematic help procedure; this consisted in reading the problem once again sentence by sentence and asking the child after each sentence to represent the situation with the manipulatives. At the end of the interview the child was confronted with an unsolvable word problem and was also asked to construct a word problem

himself. The individual interviews were videotaped. The data were submitted to a quantitative and a qualitative analysis.

3.2. Response patterns

A first test of the adequacy and the validity of Riley et al.'s computer models consists in comparing the theoretically identified patterns of performance on the change problems with the results of the thirty first graders on the distinct problems types during the three interviews in our study. However, we cannot test all the predictions that derive from the computer models, because we administered only four of the six types of change problems, namely change 1, 2, 3, and 6. In what follows we will therefore disregard the change problems 4 and 5.

Riley et al.'s (1983) model that simulates the lowest level of skill solves the change problems 1 and 2. Our finding that the thirty children solved correctly the change 2 problem already in the beginning of the school year is in accordance with this prediction. However, during the first session in the beginning of the school year there were five children who did not succeed in solving the change 1 problem, even not when the systematic help procedure was applied; the computer model cannot account for this finding. Later on a qualitative analysis of those pupil's errors will show that they are indeed due to factors that are not taken into account in the computer model (see 3.5).

Riley et al. (1983) assume that more knowledge and skills are required to solve change 3 problems than for change 1 and 2 problems. According to their analysis change 3 can only be solved correctly by model 2 representing skill level 2, whereas change 1 and 2 are already mastered by model 1 (see Table 2). This prediction is confirmed by our data: no one of our children who answered change 3 correctly failed on change 1 or 2.

The analysis by Riley et al. (1983) implies also that change 6 problems are mastered later than change 3 problems (see Tabel 2). In the first session at the beginning of the school year we did not administer the change 6 problem to those children who failed on change 3. Consequently, this prediction about the sequence in which change 3 and 6 can be solved, can only partly be verified using our data. The results are in line with the prediction: (1) over the three session we found only two cases in which a pupil solved change 6 correctly while failing on change 3; (2) during each of the three sessions there were a number of pupils - respectively three, five, and again five - who solved change 3 correctly but failed on change 6. However, the following remark is worth making. During each of the three

sessions the number of children who performed well on change 3 and failed on change 6 was much smaller than those who solved both problems correctly, namely respectively 12, 23, and 24. Although this finding is not in conflict with the prediction of Riley et al. (1983), it raises nevertheless the question concerning the reality and the importance of the distinction between the levels of skill 2 (solving the change problems 1, 2, 3, and 4) and 3 (solving the six types of change problems) in their theoretical analysis. In this respect we refer to the alternative view of Lindvall & Tamburino (1981, p. 18) who distinguish only two levels in the development of children's problem-solving skills on change problems: at level 1 the child can solve only the change 1, 2, and 4 problems; at level 2 all change problems are mastered. These investigators base their view on a study in which a group of 66 kindergarten children did solve change problems with the start set unknown (change 5 and 6) equally well as change 3 problems.

As we have mentioned in the overview of Riley et al.'s (1983) theoretical analyses, these authors hypothesize that change 6 problems can only be solved by re-representing them in terms of the combine schema. Consequently, they predict that children can cope with change 6 problems only when they master also combine 2 problems (see Table 1). Our data are not in accordance with this prediction. In each of the three sessions a number of children, respectively five, four, and one, performed well on change 6 while failing on the combine 2 problem; and in nine of these ten cases the child seemed to solve the problem with understanding. Moreover, we found that during the first two sessions a lot of children who did not yet perform well on a Piagetian class inclusion task nevertheless solved correctly the difficult change 6 problem (Verschaffel, 1984, p. 354). Both findings raise serious doubts concerning the viewpoint of Riley et al. (1983) that change 6 problems can only be solved after re-representing them in terms of the combine schema. In this connection we mention that in their computer models Briars & Larkin (1982) have implemented an alternative view of the competent problem-solving process, which does not require the transformation from a change to a combine schema.

In summary, the preceding analysis of the response patterns of the individual children in our study shows that, although our findings support a great number of predictions of the computer models concerning the sequence in which the different problem types are mastered, there are also results that are not in agreement with the hypothetic sequence.

3.3. Problem representations of the good performers

In their theoretical analysis Riley et al. (1983) also give a detailed hypothetic description of the content and the organization of the problem representations at the different levels of problem-solving skill. Two tasks in our individual interview yielded data on children's problem representations, namely retelling the story and representing it materially with puppets and blocks. A qualitative analysis of the retell protocols and the material representations of the good performers in our study allows us to verify to what degree their view of the problem situation corresponds to the problem representation constructed by the computer model. Our data show that both representations do certainly not always coincide. We will illustrate this for the change 1 problem in our study; a more extensive discussion of the data is given in Verschaffel (1984).

Riley et al. (1983) assume that for problems like our change 1 problem ("Pete had 3 apples; Ann gave Pete 5 more apples; how many apples does Pete have now?") a representation is constructed in terms of the change schema which is composed of three sets ; a start set and a change set involving the first and the second given number respectively, and a result set involving the unknown quantity. The retell protocols and the material representations of many children who solved change 1 correctly are in line with the preceding hypothesis. Those retell protocols are very similar to the verbal text of the problem, and in rendering the story with puppets and blocks these children carried out precisely the same material actions as described in Riley et al.'s computer program : first they took three blocks and put them with the puppet which represents Pete; then they added five more blocks to this set of three.

However, not all the good performers retold and materialized the change 1 problem as described by Riley et al. (1983). In the retell protocol from ten pupils there is one more sentence than in the original problem text. In this sentence those children state that initially Ann had also available a specified or unspecified number of apples. As an illustration we give one example registered during the third session: "Ann had 5 apples and Pete had 3 apples; and Ann gave 5 apples to Pete. How many apples does Pete have now?".

Most children who retold the problem in this way also played the story differently with the manipulatives. To start with they did not only put blocks with the puppet representing Pete, but also with Ann; then they moved five blocks from Ann towards Pete. In their comments those children oftentimes did not only refer to the apples that Ann had in the beginning,

but also to those that she still had available after giving five apples to Pete. The following protocol taken from the second session illustrates this obviously.

Protocol 1

I (Interviewer): Can you play the story with puppets and blocks ?

C (Child) : (Takes 3 blocks, puts them with the puppet which represent Pete and says at the same time:) Pete had 3 apples.
(Then the child takes 5 blocks and puts them with Ann; next she moves them from Ann towards Pete while she says:) and Ann gave 5 blocks away.
(The child now looks to Ann and says:) and now Ann has no more apples.
(Looking to Pete she says:) and now Pete has 8 apples.

This empirical data lead us to the hypothesis that some children construct a more elaborated representation of change 1 problems than the elementary change schema described by Riley et al. (1983). Starting from the verbal text in which only three quantities are mentioned, they seem to construct a problem representation composed of five sets instead of three : (1) a set with the number of apples which Pete had available initially; (2) a set with the number of apples which Ann had at first; (3) a set with the number of apples transferred from Ann to Pete; (4) a set with the number of apples which Pete has at the end; (5) a set with Ann's final number of apples. The problem representation of change 1 in terms of this more elaborated change schema is given in Figure 5.

Insert Figure 5 about here

Taking into account that similar data have also been obtained for other change problems (see Verschaffel, 1984), our findings allow us to conclude that not all children who solve a problem correctly, construct a representation as predicted by the theoretical analysis of Riley et al. (1983).

3.4. Solution strategies of the good performers

To assess the solution strategy of a problem, the interviewer observed closely the child's spontaneous material and/or verbal solution actions; furthermore the pupil was asked to comment on his solution procedure. Our classification schema for solution strategies is based on the one developed

by Carpenter & Moser (1982). One dimension of their scheme is the level of internalization of the strategies in which they distinguish three levels: (1) material strategies based on direct modeling of the problem situation with fingers or physical objects; (2) verbal strategies based on the use of counting sequences; (3) mental strategies based on recalled number facts (Carpenter & Moser, 1982, p. 14). On the basis of the recent literature and the results of the present study we were able to supplement and refine the Carpenter & Moser scheme (De Corte & Verschaffel, 1984; Verschaffel, 1984). Here we will only report some interesting findings on children's strategy use in view of our discussion of Riley et al.'s (1983) theoretical analysis.

In the first place we found an obvious development in the level of internalization of the appropriate solution strategies: in the beginning of the school year the children solved the word problems mainly by using material and verbal strategies; as the school year advanced the number of material and verbal strategies decreased while the mental strategies increased constituting two thirds of the appropriate strategies in June.

Secondly, we established within each of the three internalization levels a rather large variety of solution methods. It is noteworthy in this respect that at each level we found a strong relationship between the semantic structure of addition and subtraction word problems and the nature of the strategies used by the children to solve them. More specifically, the different problem types were solved most frequently with the strategy that corresponds most closely to the semantic structure underlying the problem. To illustrate this important result we refer to the mental solution strategies for the two subtraction problems with a change schema administered in our study. The data were obtained during the third session at the end of the school year. The pupils who solved the change 2 problem ("Pete had 12 apples; he gave 4 apples to Ann; how many apples does Pete have now?") mentally, used mainly direct subtractive strategies: the answer is found by subtracting the smallest given number from the largest one. On the contrary the change 3 problem ("Pete had 5 apples; Ann gave him some more apples; now Pete has 14 apples; how many apples did Ann give to him?") was solved very rarely with a direct subtractive strategy; most children operating at the mental level applied an indirect addition strategy: one determines what quantity the smallest given number must be added with to obtain the largest one.

The theoretical analysis of Riley et al. (1983) cannot account for the great majority of our data concerning the variety and the development in children's solution strategies on simple word problems, and of the influence of the task characteristics on the use of strategies. Indeed, because

the computer models based on this theoretical analysis solve word problems using blocks, they can only account for data of children who operate at the lowest level of internalization. A comparison of the strategies implemented in the computer models with those we observed, must therefore necessarily be restricted to the material level. Here too we discovered some divergencies between children's material solution strategies and the actions implied in the computer models. We describe two examples to illustrate this statement.

During each of the three sessions four pupils solved our change 1 problem ("Pete had 3 apples; Ann gave Pete 5 more apples; how many apples does Pete have now?") in the following way. Just as the computer model they first constructed a set of three blocks. However, instead of adding five blocks to this set, as the computer program does, these children made a second, separate set containing either the same or a larger number of blocks than indicated by the second quantity in the problem text. Then they removed five blocks from this set and added them to the set of three constructed first. Finally, the total number of blocks in this new set was counted and the result was given as the answer. It is very likely that these children had represented the change 1 problem in terms of the more elaborated change schema described above (see section 3.3.).

Most children who solved the change 6 problem ("Pete had some apples; he gave 3 apples to Ann; now Pete has 5 apples; how many apples did Pete have in the beginning?") at the material level, performed the same actions as the computer model of Riley et al. (1983): they made two sets of blocks corresponding to the two given numbers in the problem (3 and 5); then they counted the total number of blocks. However, five children used a different strategy. To start with they took an arbitrary number blocks; then they removed three blocks from the initial set; next they removed more blocks from the initial set or added blocks until a set of five was obtained; finally the blocks of the two sets were added and the result, namely eight, was given as the answer. Gwennie's protocol illustrates this procedure.

Protocol 2

(Gwennie had heard the problem twice and she retold it wrongly each time. Then the interviewer read the story once more and asked her to solve it.)

C: (Whispering:) Pete had a little bit (takes 5 blocks)

(She removes 3 from these blocks and puts them aside while she says:) He gave 3 away.

(Whispering:) He had 5 left (adds blocks to the original set until it contains 5 blocks)

I: And?

C: I'm still thinking how I can find it.

I: The question was: "How many apples did Pete have in the beginning?"

C: (Counts the group of 3 and 5 blocks and says:) 8

I: How did you find this answer.

C: First I have given 3 away; then I have put the 5 that he had at the end; finally I have added them up.

The difference between the procedure used by these children and the computer model is obvious: in response to the first sentence of the problem in which an unspecified quantity is introduced, the children put out an arbitrary number of blocks, while the computer program does nothing. According to Lindvall & Tamburino (1981, p. 18) this strategy applied by the pupils explains why they did not find a significant difference in difficulty level between change problems with the start set unknown and those with the change set unknown; on the contrary, Riley et al. (1983) predict that start unknown problems are more difficult (see also section 3.2.).

3.5. Errors on simple addition and subtraction problems

On the basis of Riley et al.'s (1983) theoretical analysis of the development of problem-solving skills predictions can also be made concerning children's errors on simple word problems (see Table 2). A comparison of these predictions with our empirical data reveals that the descriptions and the interpretations of errors implied in the analysis of Riley et al. (1983), cannot account for the observed diversity in children's errors nor for the variety in their origin. In the first place we have found for different problem categories, significant numbers of errors that are not at all mentioned in the computer models. In the second place we have discovered for those error types predicted by the models, origins that are substantially different from those implied in Riley et al.'s theoretical analysis. We will verify both statements with respect to the most frequently occurring error type on change problems, namely answering with one of the given numbers, either the largest (LGN) or the smallest given number (SGN).

3.5.1. LGN- and SGN-errors on change 1 and change 6 problems

The change 1 and 6 problems were both answered by a significant number of children with one of the given numbers in the problem.
During the first session three pupils gave 5 as their answer on the change 1 problem ("Pete had 3 apples; Ann gave him 5 more apples; how many apples does Pete have now?") even after the systematic help procedure was applied. None of them had retold the problem correctly, and they gave the same wrong answer before the systematic help procedure was used. As an illustration we mention a part of an interview.

Protocol 3

I: We will now try to solve the problem together. I will read it sentence by sentence and you will play it with the puppets and the blocks.
Pete had 3 apples.

C: (Takes 3 apples and puts them with Pete.)

I: Ann gave him 5 more apples.

C: (Counts the group of 3 blocks that lies with Pete and adds 2 blocks to it while she whispers:) 4, 5.

I: How many apples does Pete have now?

C: 5.

As Table 2 shows Riley et al.'s model 1 answers change 1 problems always correctly. Consequently it does not provide an explanation for the type of errors described above. Probably these errors are due to the fact that the children misinterpret a sentence, such as "person A gives person B X more objects" as follows: the consequence of person A's action is that person B has X objects, irrespective of the number of objects that B already had. That is why after hearing the second sentence of the problem, they increase the initial set from three to five instead of adding five more blocks (Verschaffel, 1984, p. 246).
Answering with the first, at the same time the smallest given number (SGN) in the problem, was the most frequently occurring error on the change 6 problem ("Pete had some apples; he gave 5 apples to Ann; now Pete has 7 apples; how many apples did Pete have in the beginning?").
During the first, the second, and the third session respectively one, eight, and five pupils gave the SGN-answer (1). The model 1 and 2 of Riley et al. (1983) do not give a wrong answer to the change 6 problem; they do not answer it at all. Therefore they can again not account for the SGN-er-

ror observed in our study. Our data, either the retelling protocol or the material solution actions, allow us to give an explanation for this error. Ten children gave the SGN-answer immediately after the interviewer had read the problem, or after they had retold it. When asked by the interviewer if they had carried out some quantitative action to find the solution their answer was negative, claiming that this was not necessary as the solution was given in the statement of the problem. All these children retold the problem wrongly and the retelling protocols confirm that their incorrect problem representation involved the information that Pete initially had five apples. We give one example.

Protocol 4

I: (Rereads the problem and asks the child again to retell it.)

C: Pete had 5 apples; now he has 7 apples; how many apples did he have first?

I: Can you solve the problem?

C: Yes: 5.

The remaining four pupils who gave the SGN-answer, continued to do so even when the systematic help procedure was used. Their material actions were as follows. The first sentence of the problem did not elicit any reaction. After the second sentence was read, they took five blocks and put them with Ann either directly or by moving then from Pete towards Ann. In reaction to the third sentence they put seven blocks with Pete. Finally when hearing the question they counted the blocks that lay with Ann and thus answered with the first given number, namely five.

Taking into account our data it is not surprising that those children gave the smallest given number as their solution for the problem. Indeed, their problem representation contained the following two elements of information: Pete had initially five apples and the question is to find out how many apples Pete had in the beginning. Of course, the question raises: why did those children think that Pete had initially only the apples that he gave to Ann and not those that are mentioned in the third sentence? A plausibel interpretation is that the inappropriate problem representation of those pupils was induced by the very condensed, in a certain sense even ambiguous formulation of the problem. For, the problem text as read by the interviewer does not mention explicitly that the apples that Pete had at the end form part of his initial amount of apples. Our change 6 problem would

therefore be more clear when stated as follows: "Pete had some apples; he gave 5 of these apples to Ann; then he still had 7 apples left; how many apples did Pete have in the beginning?". The results of a recent study support this interpretation (De Corte, Verschaffel & De Win, in press). This investigation shows that rewording verbal problems in such a way that the semantic relations are made more explicit, has a facilitating effect on the construction of an appropriate mental representation by young children.

3.5.2. Origins of LGN-errors on change 3 problems

As the computer models developed by Riley et al, (1983) do not make the LGN- and SGN-errors on the change 1 and 6 problems, they can of course not give a satisfactory explanation for these mistakes. With respect to the change 3 problem however, the most frequently observed error in our study, namely the LGN-error corresponds to the wrong answer predicted by Riley et al.'s model 1. In section 2 we have already described model 1's erroneous procedure on change 3 problems; such as: "Pete had 3 apples; Ann gave Pete some more apples; now Pete has 10 apples; how many apples did Ann give him?" In short model 1 acts in the following way: after the first sentence it makes a set of three blocks; the second sentence does not give rise to any change in the model's problem representation; in reaction to the third sentence the model adds blocks to the initial set until there are eight blocks total; when finally the question is asked model 1 counts the total number of blocks and gives ten as its answer. Riley et al. (1983) explain model 1's LGN-error as follows: (1) at the moment of asking the question the start set and the change set are not distinguished in the model's external representation with the blocks; (2) because the model does not master the general change schema, it has not available a global internal representation in which the known and unknown quantities are appropriately interrelated.

During the first, the second, and the third interview in our study respectively nine, seven, and two children answered the change 3 problem with the LGN. However, the qualitative analysis of our data revealed that the majority of these LGN-errors, namely 15 out of the 18, have different origins than the theoretical analysis by Riley et al. (1983) describes.

In the three sessions, respectively four, one, and one child gave the LGN-answer because their problem representation included the information that Ann gave Pete ten more apples, and not that she gave him more apples until he had ten total. We mention two protocols in which this faulty representation is shown very obviously: the first protocol contains retel-

ling data, while the second one reports a child's material actions.

Protocol 5

I: (Reads the problem).

C: 10

I: Can you retell the problem?

C: Pete had 3 apples; Ann gave him 10 more apples; how many apples did Ann give him?

Protocol 6

I: I will read the problem once more, and now you may solve it immediately. (Rereads the problem.)

C: (Takes 3 blocks and puts them with Pete; takes another ten blocks and puts them with Ann; moves Ann's ten blocks towards Pete while saying:) And then Pete got ten.

I: How many apples did Ann give to Pete?

C: 10

I: Let us now try to solve the problem together. I will read it sentence by sentence, and you will play it with the puppets and the blocks. Pete had 3 apples.

C: (Takes 3 blocks and puts them with Pete.)

I: Ann gave him some more apples.

C: (Takes 2 blocks and puts them with Ann.)

I: Now Pete has ten apples.

C: (Moves first the two blocks from Ann to Pete, and puts then 8 other blocks with Pete, while counting:) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

I: How many apples did Ann gave him?

C: 10.

It is again not at all surprising that children who had such an inappropriate problem representation, answered with the largest given number. Indeed, their representation contained the information that Ann gave Pete 10 apples, and at the same time the question is asked how many apples she gave him. From our data we cannot derive with certainty how the faulty information element originated in the problem representation. Possibly the children confused the expressions "give X more" and "give more till X".

Nine other children - three during the first and seven during the second interview - gave also the LGN-answer, although they had a correct representation of the main given elements in the problem. However, they did not have an appropriate representation of the unknown: they thought that the question asked for the number of apples that Pete had available at the end instead of the additional number that Ann gave him. Here again the LGN-answer derived directly from children's faulty problem representation which included at the same time the following elements of information: (1) at the end Pete has 10 apples; (2) the question asks for Pete's final number of apples. The following data support this interpretation. (1) These children gave the LGN-answer after retelling the story incompletely or as follows: "Pete had 3 apples; Ann gave Pete some more apples; now Pete has 10 apples; how many apples does Pete have now?" (2) In contrast with most pupils of the previous group of LGN-solvers, all nine children succeeded afterwards to solve the problem correctly. Mostly it was sufficient to repeat the verbal text or even only the question to elicit the correct answer.

We accentuate that the preceding two process descriptions of the LGN-answer on the change 3 problem differ substantially from the one in the computer model of Riley et al. (1983). A first group of six children answered with the LGN because they constructed a faulty problem representation: they considered the largest given number as the change set and not as the result set. The second group had an appropriate representation of the given elements in the problem, but not of the unknown.

4. Discussion

In this contribution we have presented and used data of a longitudinal study to test various predictions derived from the computer models of elementary arithmetic word problem solving developed by Riley et al. (1983). More specifically we have focused on the following aspects of Riley et al.'s analysis of the solution processes of the so-called change problems: the answer patterns of the individual children on the different types of change problems, the appropriate problem representations, the correct solution strategies, and the nature and origin of children's errors. Although a number of results confirm several predictions and implications of the computer models, we have also important findings that are not in agreement with the models, and give rise to modifications and completions of the underlying theory of children's problem-solving processes. In our view several factors should be considered to explain why the computer models cannot account for a lot of our findings.

In the first place those models are strongly based on a rational analysis of the simulated cognitive processes, and less on empirical data. Moreover, the "roughness" of those data contrasts rather sharply with the fine-grain analysis of the cognitive processes and structures implied in these models. This applies, for example, to Riley et al.'s (1983) assumption that a competent problem-solving process of change 5 and 6 problems involves a transformation in terms of the combine schema.

A second point relates to the specificity of Riley et al.'s (1983) computer models: they simulate children who solve certain types of problems by constructing, manipulating, and counting sets of blocks in reaction to the sequence of sentences in the verbal text. Their descriptive and explanatory value decreases however, as soon as the formulation and the mode of presentation of the problems do not coincide completely with the corresponding aspect in those computer models.

Finally, the text-processing component, i.e. the description of the variables and processes that contribute to the construction of an appropriate representation of the problem text, is not sufficiently elaborated in Riley et al.'s (1983) analysis. Greeno himself has pointed this out (Kintsch & Greeno, in press, p. 2). Therefore it is not very surprising that several appropriate and inappropriate problem representations are totally lacking in the computer models.

The preceding criticisms do not at all imply that we deny the value of computer simulation as a technique for research on problem solving. Because of its powerful capacity to process information quickly and in a controllable way, the computer is a very appropriate device to model cognitive structures and processes underlying intellectual performances. A computer program represents those structures and processes explicitly and unambiguously, and therefore it is an excellent starting point to generate hypotheses that can and must be verified empirically.

In our study we have used self-report techniques and behavior observation to test a series of predictions derived from Riley et al.'s (1983) models. Using such qualitative empirical data to test the adequacy and validity of computer models appears to be an interesting form of interaction between different research and assessment methods in studying cognitive processes. It is obvious, however, that other data-gathering techniques can also be applied, such as response latencies, eye-movement registration, etc. On the

basis of empirical verifications, the initial theoretical analysis of problem solving and the corresponding computer models can be modified, completed, and /or refined. The concurrent application of a variety of research techniques for the study of the same topic, either by one team of investigators or in different centers that exchange regularly their data and findings, corresponds to what one of the present authors (De Corte, 1984) has called the "broad-spectrum viewpoint" concerning research methodology.

Note

(1) That during the first session only one child answered with the smallest given number is undoubtedly due to the fact that at that time only half of the thirty children were administered this problem.

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Table 1. Overview of the fourteen types of simple addition and subtraction word problems distinguished by Heller & Greeno (1978)

Problem type	Example	Direction	Unknown
Change 1	Joe had 3 marbles. He found 5 more marbles. How many marbles did Joe have then?	increase	result set
Change 2	Joe had 8 marbles. He lost 5 marbles. How many marbles did Joe have then?	decrease	result set
Change 3	Joe had 3 marbles. He found some more marbles. Then he had 8 marbles. How many marbles did Joe find?	increase	change set
Change 4	Joe had 8 marbles. He lost some marbles. Then he had 3 marbles. How many marbles did Joe lose?	decrease	change set
Change 5	Joe had some marbles. He found 5 marbles. Then he had 8 marbles. How many marbles did Joe have to begin with?	increase	start set
Change 6	Joe had some marbles. He lost 5 marbles. Then he had 3 marbles. How many marbles did Joe have to begin with?	decrease	start set
Combine 1	Joe has 3 marbles. Tom has 5 marbles. How many marbles do they have altogether?	-	superset
Combine 2	Joe and Tom have 8 marbles altogether. Tom has 5 marbles. How many marbles does Joe have?	-	subset
Compare 1	Joe has 3 marbles. Tom has 8 marbles. How many more marbles does Tom have than Joe?	more	difference set
Compare 2	Joe has 8 marbles. Tom has 3 marbles. How many fewer marbles does Tom have than Joe?	less	difference set
Compare 3	Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?	more	compared set
Compare 4	Joe has 8 marbles. Tom has 5 less marbles than Joe. How many marbles does Tom have?	less	compared set
Compare 5	Tom has 8 marbles. He has 5 more marbles than Joe. How many marbles does Joe have?	more	referent set
Compare 6	Tom has 3 marbles. He has 5 less marbles than Joe. How many marbles does Joe have?	less	referent set

Table 2. Levels of performance on change problems
(Riley et al., 1983)

Examples of problems	Problem type	Levels of performance 1	2	3
1. Joe had 3 marbles. Then Tom gave him 5 more marbles. How many marbles does Joe have now?	Change 1	+	+	+
2. Joe had 8 marbles. Then he gave 5 marbles to Tom. How many marbles does Joe have now?	Change 2	+	+	+
3. Joe had 3 marbles. Then Tom gave him some more marbles. Now Joe has 8 marbles. How many marbles did Tom give him?	Change 3	"8"	+	+
4. Joe had 8 marbles. Then he gave some marbles to Tom. Now Joe has 3 marbles. How many marbles did he give to Tom?	Change 4	+	+	+
5. Joe had some marbles. Then Tom gave him 5 more marbles. Now Joe has 8 marbles. How many marbles did Joe have in the beginning?	Change 5	"5"	"5"	+
6. Joe had some marbles. Then he gave 5 marbles to Tom. Now Joe has 3 marbles. How many marbles did Joe have in the beginning?	Change 6	NA*	NA	+

* = No answer.

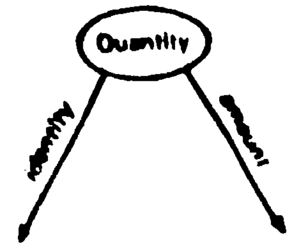

Figure 1. Schema for representing quantitative information of Riley et al.'s (1983) model 1

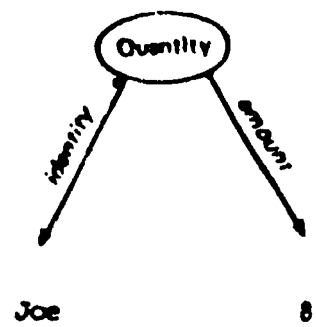

Figure 2. Mental representation constructed by Riley et al.'s (1983) model 1 of the change 3 problem in Table 2

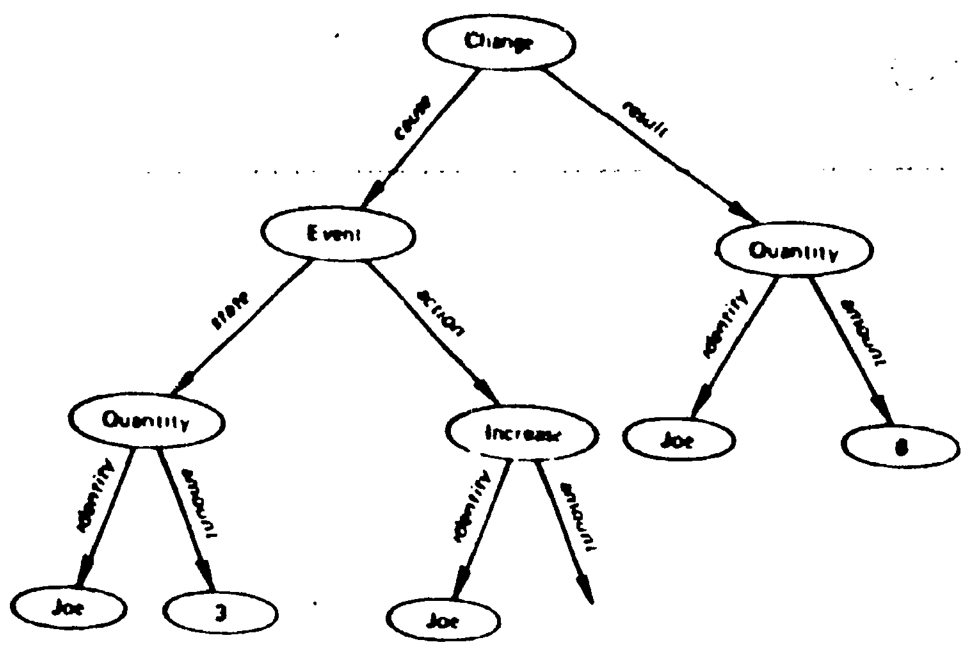

Figure 3. Mental representation constructed by Riley et al.'s (1983) model 2 of the change 3 problem in Table 2

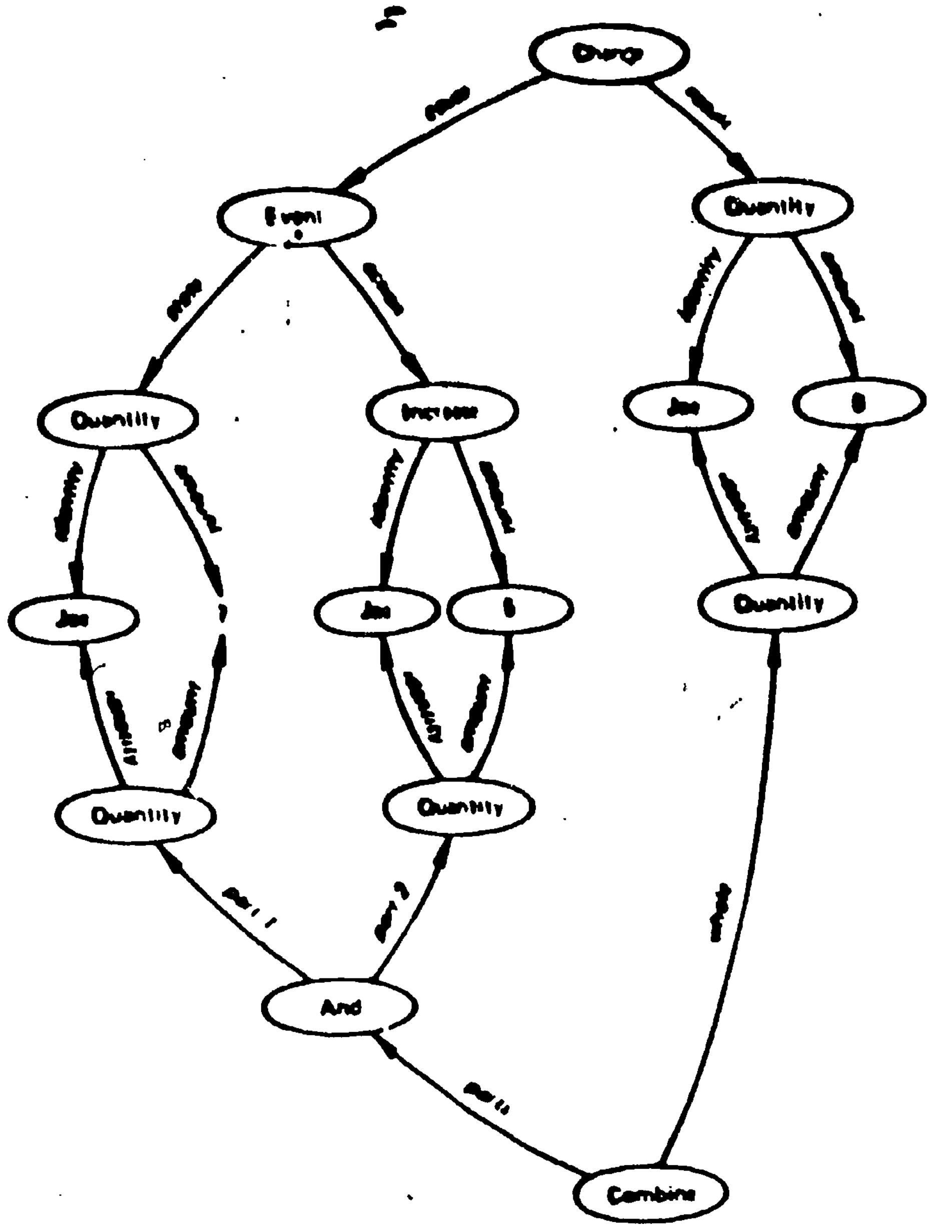

Figure 4. Transformation by Riley et al.'s (1983) model 3 of the initial representation of the change 5 problem in Table 2 in terms of the combine schema

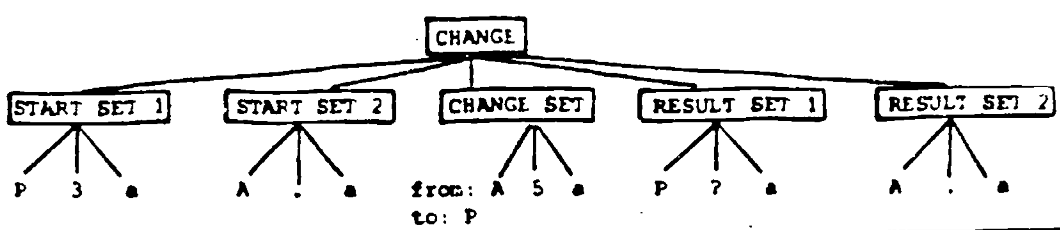

Figure 5. Representation of the change 1 problem in terms of the elaborated change schema